Multiple Criteria Group Decision Making based on Interval-valued Pythagorean Fuzzy Hamacher Weighted Averaging Operator

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Abstract: By integrating the algebraic aggregation operators and Einstein aggregation operators, an interval-valued pythagorean fuzzy Hamacher weighted averaging operator are proposed. Then, some desirable properties of these operators are considered. Moreover, an illustrative example is provided to demonstrate their practicality and effectiveness of the proposed method.

1. Introduction

Multiple criteria group decision making (MCGDM) is one of the main tasks of the decision theory. As the fuzziness and uncertainty exists in the presentation of data information in the decision-making process, Atanassov [1] initiated the intuitionistic fuzzy set (IFS) which is characterized by a membership degree and a nonmembership degree, IFS have two memberships which reduce the fuzziness. Atanassov and Gargov [2] further proposed the interval-valued intuitionistic fuzzy set (IVIFS) in which the membership degree and nonmembership degree are extended to interval numbers. However, there exist some situations that the sum of membership degree and nonmembership degree exceeds 1. This is not within the researching aim of IFS(IVIFS) theory. By losing this limitation that the sum of membership degree and nonmembership degree is less than 1,Yager [3] proposed the concept of Pythagorean fuzzy set(PFS) which is characterized by the requirement that its square sum of membership degree and nonmembership degree is not greater than 1.Later,Garg [4] extended the IVIFNs to the IVPFNs by the condition that the square sum of the two upper-memberships of the two interval is less than 1.Since then, many decision making problems relating to the PFS(IVPFS) theory have been made. In addition to the decision-making methods above, several aggregated [5] operators-based approaches have been proposed.

The rest of this paper is arranged as follows. In Section 2, we briefly review some basic concepts of IVPFNs and Hamacher t−conorm and t−norm. In Section 3, we define Hamacher operations on IVPFNs and develop some Hamacher arithmetic aggregation operators based on IVPFNs operator. In Section 4, an example is presented to illustrate the application of these methods.

2. Preliminaries

2.1 Interval-Valued Pythagorean Fuzzy Set.

Definition 1. Let X be a set, an interval valued Pythagorean fuzzy set (IVPFS) A in X is defined as

$$A=\{\langle x,\mu_A(x),\nu_A(x)\rangle \mid x\in X\}$$

Where $\mu_A(x)$ and $\nu_A(x)$ with the condition $0\le \sup(\mu_A(x)+\nu_A(x))\le 1,$ the intervals $\mu_A(x)$ and $\nu_A(x)$ represent, respectively, the membership degree and nonmembership degree of the element x to the set A. For each $x\in X$, and $\mu_A(x)$ and $\nu_A(x)$ are closed intervals and their lower and upper end points are, respectively, denoted by. $\mu_{AL}(x)$ and $\mu_{AU}(x)$, $\nu_{AL}(x)$ and $\nu_{AU}(x)$ .and $0\le \mu^2{}_{AU}(x)+\nu^2{}_{AU}(x)\le 1.$Thus, an IVPFS A in X is expressed by $A=\{\langle x\mid[\mu_{AL}(x),\mu_{AU}(x)],[\nu_{AL}(x),\nu_{AU}(x)]\rangle \mid x\in X\}$, Where $0\le \mu^2{}_{AU}(x)+\nu^2{}_{AU}(x)\le 1.$

Definition 2. Let $\alpha=\langle[a,b],[c,d]\rangle$.

The score of α is defined as S (α)=(a^2- b^2+ c^2- d^2)/2

2.2 Interval-Valued Hamacher t−Norm and t−Conorm

Union and intersection are two basic operations of fuzzy set, as a generalization of the two operations, *t*−norm and *t*−conorm are developed in fuzzy set theory .As a special cases of *t*−norm and *t*−conorm, Hamacher operations which consist of Hamacher sum and Hamacher product are introduced by Hamacher, they are put as follows

$$T(x,y)=\frac{xy}{r+(1-r)(x+y-xy)},S(x,y)=\frac{x+y-xy-(1-r)xy}{1-(1-r)xy}$$

Definition 3.

Let $\alpha=\{\langle x\,|\,[\mu_{\alpha L}(x),\mu_{\alpha U}(x)],[\nu_{\alpha L}(x),\nu_{\alpha U}(x)]\rangle\,|\,x\in X\},\beta=\{\langle x\,|\,[\mu_{\beta L}(x),\mu_{\beta U}(x)],[\nu_{\beta L}(x),\nu_{\beta U}(x)]\rangle\,|\,x\in X\},n>0$

be any two IVPFNS, then, the generalized intersection and union of α,β are defined as follows:

$$(1)\alpha\otimes\beta=\langle[(S((\mu_{\alpha L}(x))^2,S((\mu_{\beta L}(x))^2)^{1/2},(S((\mu_{\alpha U}(x))^2,S((\mu_{\beta U}(x))^2)^{1/2}],$$
$$[(T((\nu_{\alpha L}(x))^2,T((\nu_{\beta L}(x))^2)^{1/2},T((\nu_{\alpha U}(x))^2,T((\nu_{\beta U}(x))^2)^{1/2}]\rangle,$$

$$(2)\alpha\oplus\beta=\langle[(T((\mu_{\alpha L}(x))^2,T((\mu_{\beta L}(x))^2)^{1/2},(T((\mu_{\alpha U}(x))^2,T((\mu_{\beta U}(x))^2)^{1/2}],$$
$$[(S((\nu_{\alpha L}(x))^2,S((\nu_{\beta L}(x))^2)^{1/2},S((\nu_{\alpha U}(x))^2,S((\nu_{\beta U}(x))^2)^{1/2}]\rangle,$$

$$(3)n\alpha=\Big\langle\Big[\sqrt{\frac{(1+(r-1)(\mu_{\alpha L}(x))^2)^n-(1-(\mu_{\alpha L}(x)^2))^n}{(1+(r-1)(\mu_{\alpha L}(x)^2)^n+(r-1)(1-(\mu_{\alpha L}(x)^2)^n}},\sqrt{\frac{(1+(r-1)(\mu_{\alpha U}(x)^2)^n-(1-(\mu_{\alpha U}(x)^2)^n}{(1+(r-1)(\mu_{\alpha U}(x)^2)^n+(r-1)(1-(\mu_{\alpha U}(x)^2)^n}}\Big],$$
$$\Big[\frac{\sqrt{r}(\nu_{\alpha L}(x))^n}{\sqrt{(1+(r-1)(1-(\nu_{\alpha L}(x))^2)^n+(r-1)((\nu_{\alpha L}(x)^2)^n}},\frac{\sqrt{r}(\nu_{\alpha U}(x))^n}{\sqrt{(1+(r-1)(1-(\nu_{\alpha U}(x)^2))^n+(r-1)((\nu_{\alpha U}(x)^2)^n}}\Big]\rangle,$$

$$(4)\alpha^n=\left\langle\Big[\frac{\sqrt{r}(\mu_{\alpha L}(x))^n}{\sqrt{(1+(r-1)(1-(\mu_{\alpha L}(x))^2)^n+(r-1)((\mu_{\alpha L}(x)^2)^n}},\frac{\sqrt{r}(\mu_{\alpha U}(x))^n}{\sqrt{(1+(r-1)(1-(\mu_{\alpha U}(x)^2))^n+(r-1)((\mu_{\alpha U}(x)^2)^n}}\Big]\right\rangle,$$
$$\Big[\sqrt{\frac{(1+(r-1)(\nu_{\alpha L}(x))^2)^n-(1-(\nu_{\alpha L}(x)^2))^n}{(1+(r-1)(\nu_{\alpha L}(x)^2)^n+(r-1)(1-(\nu_{\alpha L}(x)^2)^n}},\sqrt{\frac{(1+(r-1)(\nu_{\alpha U}(x)^2)^n-(1-(\nu_{\alpha U}(x)^2)^n}{(1+(r-1)(\nu_{\alpha U}(x)^2)^n+(r-1)(1-(\nu_{\alpha U}(x)^2)^n}}\Big]\Big\rangle$$

3. The Proposed Hamacher Aggregating Operators

Definition 4. Let Θ be the set of IVPFNs, $\alpha_i=\langle[\mu_{\alpha_i L}(x),\mu_{\alpha_i U}(x)],[\nu_{\alpha_i L}(x),\nu_{\alpha_i U}(x)]\rangle$ be the set of IVPFNs, where $w=(w_1,w_2,\cdots,w_n)^T$ is the weight vector of $\alpha_i(i=1,2,\cdots,n)$ then, with $w_1\in[0,1]$ and $\sum_{i=1}^{n}w_i=1,$ and let *IVPFHWA*: $\Theta^n\to\Theta$,if $IVPFHWA_w(\alpha_1,\alpha_2,\cdots,\alpha_n)=w_1\alpha_1\oplus w_2\alpha_2\oplus\cdots\oplus w_n\alpha_n$.

Then, IVPFHWA is called the interval-valued Pythagorean fuzzy Hamacher weighted averaging operator.

Theorem**.** Let Θ be the set of IVPFNs, $\alpha_i=\langle[\mu_{\alpha_i L}(x),\mu_{\alpha_i U}(x)],[\nu_{\alpha_i L}(x),\nu_{\alpha_i U}(x)]\rangle$ be the set of IVPFNs, where $w=(w_1,w_2,\cdots,w_n)^T$ is the weight vector of $\alpha_i(i=1,2,\cdots,n)$ then, with $w_1\in[0,1]$, $\sum_{i=1}^{n}w_i=1,$ then, the aggregating result from Definition 3 is an IVPFN.

And:

$$IVPFHWA_w(\alpha_1,\alpha_2,\cdots,\alpha_n)=$$

$$\langle[\sqrt{\frac{\prod_{i=1}^{n}(1+(r-1)(\mu_{\alpha_i L}(x))^2)^{w_i}-\prod_{i=1}^{n}(1-(\mu_{\alpha_i L}(x)^2))^{w_i}}{\prod_{i=1}^{n}(1+(r-1)(\mu_{\alpha_i L}(x))^2)^{w_i}+(r-1)\prod_{i=1}^{n}(1-(\mu_{\alpha_i L}(x)^2))^{w_i}}},\sqrt{\frac{\prod_{i=1}^{n}(1+(r-1)(\mu_{\alpha_i U}(x))^2)^{w_i}-\prod_{i=1}^{n}(1-(\mu_{\alpha_i U}(x)^2))^{w_i}}{\prod_{i=1}^{n}(1+(r-1)(\mu_{\alpha_i U}(x))^2)^{w_i}+(r-1)\prod_{i=1}^{n}(1-(\mu_{\alpha_i U}(x)^2))^{w_i}}}],$$

$$[\frac{\sqrt{r}\prod_{i=1}^{n}(\nu_{\alpha_i L}(x))^{w_i}}{\sqrt{\prod_{i=1}^{n}(1+(r-1)(1-(\nu_{\alpha_i L}(x))^2)^{w_i}+(r-1)\prod_{i=1}^{n}(\nu_{\alpha_i L}(x)^2)^{w_i}}},\frac{\sqrt{r}\prod_{i=1}^{n}(\nu_{\alpha_i U}(x))^{w_i}}{\sqrt{\prod_{i=1}^{n}(1+(r-1)(1-(\nu_{\alpha_i U}(x))^2)^{w_i}+(r-1)\prod_{i=1}^{n}(\nu_{\alpha_i U}(x)^2)^{w_i}}}]\rangle$$

4. Experiment

We utilize the proposed method to select the optimal high-tech enterprise with the lowest risk of technologic innovation from four candidate high-tech enterprises $\{A_1, A_2, A_3, A_4\}$.The criteria designed for the risk evaluation of technologic innovation are chosen as follows:C_1:Policy risk;C_2:Financial risk; C_3:Technological risk;C_4:Production risk;C_5:Market risk;C_6:Managerial risk. The three decision makers(experts) $\{E_1,E_2,E_3\}$ who are specializing in risk evaluation fields are invited to evaluate these four high-tech enterprises according to the six evaluation criteria $\{C_1,C_2,C_3,C_4,C_5,C_6\}$.The weight vector of experts is given beforehand as ϖ = (0.4,0.35,0.25),and the weight vector of criteria is w = (0.1894,0.1841,0.1361,0.1257,0.1753,0.1894).The entries values of alternatives with respect to criteria provided by the experts are assumed to be represented by IVPFNs as shown in interval-valued Pythagorean fuzzy group decision matrix which are listed in Table 1,2,3.

Table 1. Matrix given by expert E_1

Attribute	C_1	C_1	C_1
1	P ([0.8,0.9], [0.2,0.3])	P ([0.5,0.7], [0.4,0.5])	P ([0.7,0.9], [0.3,0.4])
1	P ([0.6,0.7], [0.3,0.5])	P ([0.6,0.8], [0.4,0.5])	P ([0.7,0.9], [0.2,0.3])
1	P ([0.5,0.6], [0.4,0.5])	P ([0.7,0.8], [0.4,0.5])	P ([0.6,0.7], [0.3,0.4])
1	P ([0.4,0.6], [0.3,0.5])	P ([0.8,0.9], [0.2,0.3])	P ([0.7,0.8], [0.4,0.5])
Attribute	C_4	C_5	C_6
1	P ([0.7,0.8], [0.4,0.5])	P ([0.8,0.9], [0.2,0.3])	P ([0.4,0.6], [0.5,0.7])
1	P ([0.8,0.9], [0.2,0.3])	P ([0.5,0.6], [0.1,0.3])	P ([0.8,0.9], [0.1,0.3])
1	P ([0.7,0.9], [0.1,0.3])	P ([0.4,0.6], [0.2,0.3])	P ([0.7,0.8], [0.4,0.5])
1	P ([0.6,0.8], [0.4,0.5])	P ([0.5,0.6], [0.1,0.3])	P ([0.8,0.9], [0.2,0.3])

Table 2. Matrix given by expert E_2

Attribute	C_1	C_1	C_1
1	P ([0.6,0.7], [0.2,0.3])	P ([0.7,0.9], [0.1,0.3])	P ([0.5,0.7], [0.2,0.5])
1	P ([0.5,0.7], [0.4,0.5])	P ([0.5,0.6], [0.3,0.5])	P ([0.5,0.8], [0.4,0.6])
1	P ([0.5,0.6], [0.4,0.5])	P ([0.8,0.9], [0.2,0.3])	P ([0.5,0.6], [0.4,0.5])
1	P ([0.7,0.9], [0.1,0.3])	P ([0.7,0.8], [0.4,0.5])	P ([0.6,0.8], [0.4,0.5])
Attribute	C_4	C_5	C_6
1	P ([0.8,0.9], [0.2,0.3])	P ([0.5,0.7], [0.2,0.4])	P ([0.5,0.7], [0.4,0.5])
1	P ([0.5,0.6], [0.1,0.3])	P ([0.6,0.8], [0.4,0.5])	P ([0.8,0.9], [0.2,0.3])
1	P ([0.6,0.7], [0.4,0.6])	P ([0.8,0.9], [0.2,0.3])	P ([0.6,0.7], [0.4,0.6])
1	P ([0.4,0.6], [0.2,0.3])	P ([0.7,0.9], [0.1,0.3])	P ([0.5,0.7], [0.4,0.5])

Table 3. Matrix given by expert E_3

Attribute	C_1	C_1	C_1
1	P ([0.8,0.9], [0.2,0.3])	P ([0.7,0.9], [0.1,0.3])	P ([0.5,0.7], [0.2,0.5])
1	P ([0.6,0.7], [0.1,0.3])	P ([0.4,0.6], [0.1,0.3])	P ([0.8,0.9], [0.2,0.3])
1	P ([0.6,0.8], [0.4,0.5])	P ([0.8,0.9], [0.2,0.3])	P ([0.7,0.9], [0.1,0.3])
1	P ([0.8,0.9], [0.2,0.3])	P ([0.7,0.8], [0.4,0.5])	P ([0.6,0.8], [0.4,0.5])
Attribute	C_4	C_5	C_6
1	P ([0.6,0.8], [0.3,0.4])	P ([0.6,0.8], [0.4,0.5])	P ([0.6,0.7], [0.4,0.5])
1	P ([0.8,0.9], [0.2,0.3])	P ([0.7,0.8], [0.2,0.5])	P ([0.8,0.9], [0.2,0.3])
1	P ([0.6,0.8], [0.4,0.5])	P ([0.6,0.7], [0.4,0.6])	P ([0.7,0.8], [0.4,0.5])
1	P ([0.8,0.9], [0.2,0.3])	P ([0.4,0.5], [0.4,0.6])	P ([0.5,0.6], [0.4,0.5])

With the two weights ϖ and w, by aggregating the vectors, *r*=3, we get

$$r_1 = P([0.6362, 0.8038],[0.6362, 0.8038]), r_2 = P([0.6537, 0.7991],[0.2086, 0.3807])$$

$$r_3 = P([0.6456, 0.7775],[0.2991, 0.4342]), r_4 = P([0.6468, 0.7984],[0.2491, 0.3998])$$

Calculate the scores with Definition 1, we get:

$$S(r_1) = 0.4072, \mathrm{S}(r_2) = 0.4387, S(r_3) = 0.3717, S(r_4) = 0.4169$$

That is,

$$\mathrm{S}(r_2) > S(r_4) > S(r_1) > S(r_3)$$

Finally, we get $A_2 \succ A_4 \succ A_1 \succ A_3$. Consequently, the best alternative is A_3. This result is the same as the one in [6].

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